EXHIBIT AB

2

CRYSTALS, CRYSTAL GROWTH, AND NUCLEATION

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2.1. CRYSTALS

Crystals are solids in which the atoms are arranged in a periodic repeating pattern that extends in three dimensions. While all crystals are solids, not all solids are crystals. Materials that have short-range rather than long-range ordering, like glass, are noncrystalline solids. A noncrystalline solid is often referred to as an amorphous solid. Many materials can form solids that are crystalline or amorphous, depending on the conditions of growth. In addition, some materials can form crystals of the same composition but with differing arrangements of the atoms forming different three-dimensional structures. Other materials can have the same three-dimensional structure but appear different in shape when viewed under the microscope. To make sense of this, and to understand the nature of crystals and how they are identified requires some knowledge of crystals and their structure. The study of crystal structure is called *crystallography* and is described in a number of standard references (Bunn 1961; Cullity 1978). In this section, we will discuss the basics of crystals and their structure.

2.1.1. LATTICES AND CRYSTAL SYSTEMS

Crystals are solids in which the atoms are arranged in a three-dimensional repeating periodic structure. If we think of crystals in a purely geometric sense and forget about the actual atoms, we can use a concept known as a point lattice to represent the crystal. A point lattice is a set of points arranged so that each point has identical surroundings. In addition, we can characterize a point lattice in terms of three spatial dimensions: a, b, and c, and three angles: α, β, and γ. An example of a point lattice is given in Figure 2.1. Looking at Figure 2.1 we can see that the lattice is made up of repeating units that can be characterized by the three dimensions and three angles mentioned. We can arbitrarily choose any of these units, and by making use of the spatial dimensions and angles can reproduce the lattice indefinitely. The lengths and angles mentioned are known as *lattice parameters* and a single cell constructed employing these parameters is called the *unit cell.* A unit cell is shown in Figure 2.2.

There are obviously a number of different lattice arrangements and unit cells that can be constructed. It was shown, however, in 1848 by Bravais that there are only 14 possible point lattices that can be constructed. These point lattices can be divided into seven categories (crystal systems) that are shown in Table 2.1. Figure 2.3 shows all 14 of the Bravais lattices. Looking at the crystal systems we see that they are all characterized by these lattice parameters. For example, cubic systems all must have equal lengths (a = b = c) and angles equal to 90°. In addition, lattices can be classified as primitive or nonprimitive. A primitive lattice has only one lattice point per unit cell while a nonprimitive unit cell has more than one. If we look at the cubic system, a simple cubic unit cell is primitive. This is because each lattice point on a corner is shared by eight other cells so that 1/8 belongs to a single cell. Since there are eight corners, the simple cubic cell has one lattice point. Looking at a body centered cubic cell, the point on the interior is not shared with any other cell. A body centered cubic cell, therefore, has two lattice points. A face centered cubic cell has a lattice point on each face that is shared between two cells. Since there are six faces as well as the eight corners, a face centered cubic cell has four lattice points.

Another property of each crystal system that distinguishes one system from another is called symmetry. There are four types of symmetry operations: reflection, rotation, inversion, and rotation-inversion. If a lattice has one of these types of symmetry, it means that after the required operation, the lattice is superimposed upon itself. This is easy to see in the cubic system. If we define an axis normal to any face of a cube and rotate the cube about that axis, the cube will superimpose upon itself after each 90° of rotation. If we divide the degrees of rotation into 360°, this tells us that a cube has three fourfold rotational symmetry axes (on axes normal to three pairs of parallel faces). Cubes also have threefold rotational symmetry using an axis along each body diagonal (each rotation is

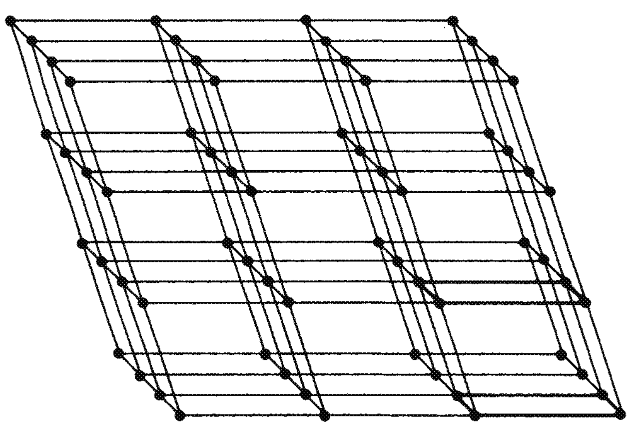

Figure 2.1 A point lattice.

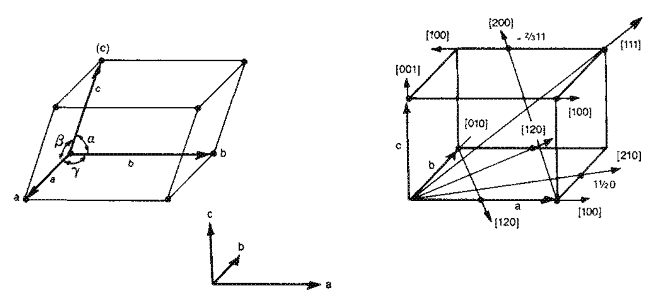

Figure 2.2 A unit cell.